the type of many of its general forms might be found in earlier examples.

Mr. Petrie then proposed that the thanks of the Academy should be given to the Rev. Richard Butler, for his kindness in sending this brooch for the inspection of the Meeting. The thanks of the Academy were accordingly voted to Mr. Butler.

Mr. Ingram read the following note on certain Properties of the Surfaces of the Second Degree.

" Mr. Salmon, Fellow of Trinity College, has given a mode of generating certain of the surfaces of the second degree, which is in a remarkable way supplementary to the *modular* method of Professor Mac Cullagh, and which has been called, for distinction's sake, the *umbilicar* method. In it the surface is had as the locus of a point moving so that the square of its distance from a fixed point is proportional to the rectangle under its distances from two fixed planes. Out of this generation arise many highly interesting properties of the surfaces in question, to some of which it is the object of the present communication to call the attention of the Academy.

" The fixed point is called the Focus of the surface, the two fixed planes the Directive Planes, and their line of intersection the Directrix.

" 1. Two right lines, reciprocal-polars with relation to the surface, meet a directive plane in two points such that the vectors drawn to them from the focus are at right angles.

" 2. A similar theorem holds for two conjugate tangents at any point of the surface.

" 3. Two right lines, reciprocal-polars with relation to the surface, seen from the pole of a directive plane, appear to cut at right angles.

" 4. Let a cone be described, passing through two plane sections of the surface; it will intersect a directive plane in a certain conic: let a second cone be described, passing through

this conic, and having its vertex at the focus ; the cyclic planes of this latter cone will pass respectively through the two right lines in which the planes of the two sections meet the directive plane.

" 5. Hence, through a right line, situated in a directive plane, draw two planes cutting the surface in two conics; a cone passing through these two curves intersects that directive plane in a certain conic : now let a cone be described, passing through this conic, and having its vertex at the focus; this latter cone will be one of revolution, and its principal plane will pass through the right line assumed in the directive plane.

" 6. Hence, a cone enveloping the surface intersects a directive plane in a certain conic; a cone passing through this conic, and having its vertex at the focus, is one of revolution; and its principal plane passes through the right line in which the plane of contact of the enveloping cone meets the directive plane.

" 7. A plane curve is traced on the surface, and through it a cone is described, having its vertex at the pole of a directive plane; this cone cuts the directive plane in a certain conic : now let a second cone be described, passing through this conic, and having its vertex at the focus; the latter cone is one of revolution, and its principal plane passes through the right line in which the plane of the original curve intersects the directive plane.

" 8. If a cone be described, having its vertex at the focus, and passing through a plane section of the surface, the cyclic planes of this cone will pass respectively through the two right lines in which the plane of the section meets the two directive planes ; and the *directive* axis of the cone will therefore be the right line drawn from its vertex to the point where the directrix is cut by the plane of the section.

" 9. Hence, any plane passing through the directrix intersects the surface in a curve such that the cone passing through it, and having its vertex at the focus, is one of revolution;

and the principal plane of that cone passes through the directrix.

" The above properties are true for *every* point on the *umbilicar* focal of the surface, with the directive planes and directrix corresponding to that point. When the two directive planes coincide, these theorems, suitably modified, reduce to known properties of the *non-modular* surfaces of revolution. For that particular case they have been demonstrated by M. Chasles, in the Transactions of the Royal Academy of Brussels (*Nouveaux Mémoires*, tom. v.) In the general shape in which they are, I believe, now for the first time* given, they appear to me of sufficient elegance to merit the attention of geometers."

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* " Since the above note was read, my attention has been directed to a paper ' On the Focal Properties of Surfaces of the Second Order,' by Dr. Booth, in the Philosophical Magazine for December, 1840. In that paper he considers as analogous to the foci in conic sections four points, which he calls the foci of the surface, situated, two by two, on the umbilical diameters, at distances from the centre equal to each other and to $u\varepsilon$, where u is the length of the umbilical semi-diameter, and $\varepsilon^2 = \frac{a^2-c^2}{a^2}$ $(a > b > c.)$ The polar planes of these points he terms the ' directrix planes' of the surface, and of these planes the two which intersect in a directrix of the principal section (a, b) are ' conjugate directrix planes.' The foci of the same section (a, b) he calls the ' focal centres' of the surface. These definitions being premised, he states the theorem, that if from any point of the surface perpendiculars be let fall on two conjugate directrix planes, the rectangle under those perpendiculars is to the square of the distance of the point from the corresponding focal centre in a constant ratio. But he does not observe the fact which gives the umbilicar generation its chief interest and value, namely, that the ' focal centre' may traverse the focal curve on which it lies, the ' directrix planes' changing along with it, while the generated surface remains unaltered. He then proceeds to state several properties of his ' focal centres' and ' directrix planes,' and among them I find those which I have marked (1), (2), (8), and (9). But these theorems are given by him only for his two ' focal centres' and his four ' directrix planes ;' whereas they are really properties of every point on the umbilicar focal of the surface, and the directive planes corresponding to such point."